CHAPTER 5

CIRCULAR MOTION; GRAVITATION

OBJECTIVES

After studying the material of this chapter, the student should be able to:

- calculate the centripetal acceleration of a point mass in uniform circular motion given the radius of the circle and either the linear speed or the period of the motion.
- identify the force that is the cause of the centripetal acceleration and determine the direction of the acceleration vector.
- use Newton's laws of motion and the concept of centripetal acceleration to solve word problems.
- distinguish between centripetal acceleration and tangential acceleration.
- state the relationship between the period of the motion and the frequency of rotation and express this relationship using a mathematical equation.
- write the equation for Newton's universal law of gravitation and explain the meaning of each symbol in the equation.
- determine the magnitude and direction of the gravitational field strength (g) at a distance r from a body of mass m.
- use Newton's second law of motion, the universal law of gravitation, and the concept of centripetal acceleration to solve problems involving the orbital motion of satellites.
- explain the "apparent" weightlessness of an astronaut in orbit.
- state from memory Kepler's laws of planetary motion.
- use Kepler's third law to solve word problems involving planetary motion.
- use Newton's second law of motion, the universal law of gravitation, and the concept of centripetal acceleration to derive Kepler's third law.
- solve word problems related to Kepler's third law.
- identify the four forces that exist in nature.

KEY TERMS AND PHRASES

uniform circular motion occurs when an object travels in a circle at constant speed.

tangential velocity refers to the direction of the velocity vector of an object as it travels in a curved path. The velocity vector is tangent to the path at each point in the curve.

tangential acceleration refers to the change in speed per unit time of an object as it travels in a curved path. The tangential acceleration is directed tangent to the path at each point.

centripetal acceleration or radial acceleration is the inward radial acceleration experienced by an object traveling in a circle. The direction of the acceleration vector is perpendicular to the direction of the velocity vector.

period (T) of the circular motion is the time required for an object to complete one revolution.

frequency of rotation (f) is the number of repetitions of the motion per unit time. The frequency is inversely related to the period of the motion.

centripetal force is the term used for the force acting toward the center of the circle. The centripetal force is not some new kind of force. The term merely means that the force is directed toward the circle's center. The force must be applied by some external agent.

Newton's universal law of gravitation is the force of attraction between any two objects due to their mass. The law states that the magnitude of the force between two particles is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

Kepler's first law states that the path of each planet about the Sun is an ellipse with the Sun at one of the focal points of the ellipse.

Kepler's second law states that each planet moves such that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time.

Kepler's third law states that the ratio of the squares of the periods of any two planets revolving about the Sun is equal to the ratio of the cubes of their average distances from the Sun.

SUMMARY OF MATHEMATICAL FORMULAS

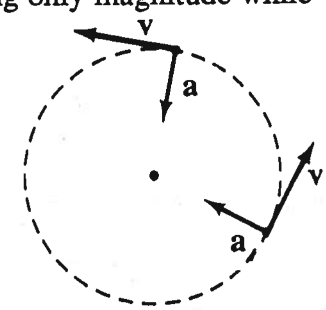
centripetal or radial acceleration	$a_R = v^2/r$	Centripetal or radial acceleration (a_R) as related to tangential velocity (v) and the radius of the circle (r).
frequency and period of rotation	$f = 1/T$ $T = 1/f$	Frequency of rotation (f) is inversely related to the period (T) of the motion.
tangential velocity	$v = 2\pi r/T$ $v = 2\pi\ r\ f$	Tangential velocity (v) as related to the radius of the circle and the period (T) or frequency (f) of the motion.
centripetal or radial acceleration	$a_R = 4\ \pi^2\ r/T^2$ $a_R = 4\ \pi^2\ r\ f^2$	Centripetal or radial acceleration (a_R) as related to the radius of the circle and the period (T) or frequency (f) of the motion.
centripetal force	$\Sigma F_R = m\ v^2/r$	Centripetal force (F_R) as related to the object's mass (m), speed (v), and the radius of the circle.

Newton's universal law of gravitation	$F = G\, m_1 m_2/r^2$	Gravitational force between two particles of mass m_1 and m_2 separated by a radial distance (r). G = universal constant = 6.67×10^{-11} N m^2/kg^2.
Kepler's third law	T^2/r^3 = constant or $T_1^2/T_2^2 = r_1^3/r_2^3$	Period of the motion (T) squared of a planet traveling about the Sun divided by the average radius (r) of its orbit cubed is a constant for all planets.

CONCEPT SUMMARY

Kinematics of Uniform Circular Motion

An object traveling in a circle at constant speed is exhibiting **uniform circular motion.** Although the object's speed is not changing, its direction of motion is and therefore its velocity is constantly changing. Note: recall that speed is a scalar quantity having only magnitude while velocity is a vector quantity having both magnitude and direction.



Since its velocity is changing, the object is undergoing accelerated motion. This acceleration is called **centripetal acceleration.** As shown in the diagram, the velocity vector at every point is directed tangent to the circle while the acceleration vector is directed toward the center of the circle. The direction of the acceleration vector is perpendicular to the direction of the velocity vector.

The magnitude of the centripetal acceleration (a_R) is directly proportional to the square of the speed and inversely proportional to the radius of the circle in which the object is traveling.

$a_R = v^2/r$

An object moving in uniform circular motion travels a distance equal to the circumference of the circle ($2\pi r$) in a time interval called the **period** (T) of the motion. The period of the motion is the time required for the motion to repeat. The equation for the speed is as follows:

$v = 2\pi r/T$

By substituting the above expression for v into the centripetal acceleration equation, it can be shown that the formula for the centripetal acceleration can be written as follows:

$a_R = (2\pi r/T)^2/r = 4\pi^2 r/T^2$

TEXTBOOK QUESTION 2. Will the acceleration of a car be the same if it travels around a sharp curve at a constant 60 km/h as when it travels around a gentle curve at the same speed? Explain.

ANSWER: The acceleration is *not* the same. The centripetal acceleration is directly proportional to the square of the speed and inversely proportional to the radius of the curve. The speed is held constant at 60 km/h but the radius of the sharp curve is less than the gentle curve. Therefore, the *smaller* the radius, the *greater* the centripetal acceleration.

Frequency of Rotation

The **frequency of rotation** (f) is the number of repetitions of the motion per unit time. For example, a stereo turntable rotating at 45 rpm has a frequency of rotation of 45 revolutions per minute.

The period of the motion is inversely related to the frequency of rotation ($T = 1/f$ or $f = 1/T$). If the frequency of rotation is 10 revolutions per second, then the period of the motion is 1/10 second or 0.10 seconds.

Since $f = 1/T$, the equation for the speed $v = 2\pi r/T$ can be written as in terms of the frequency as

$$v = 2\pi\, r\, f$$

Also, the equation for centripetal acceleration $a_R = 4\,\pi^2\, r/T^2$ can be written as $a_c = 4\,\pi^2\, r\, f^2$.

Dynamics of Uniform Circular Motion

An object traveling in a circle must have a net force acting on it to keep it in the circle. The magnitude of the object's acceleration can be determined by using Newton's second law, i.e., $a = \text{net } F/m$. For an object traveling in a circle, the acceleration is the centripetal acceleration. Since the centripetal acceleration is directed toward the center of the circle, the net force, called the **centripetal force,** must also be directed toward the center of the circle. As stated in the textbook, "This force is sometimes called a centripetal force. But be aware that 'centripetal force' does not indicate some new kind of force. The term merely means that the force is directed toward the circle's center. The force must be applied by some object."

In solving problems involving uniform circular motion, it is usually necessary to determine the nature of the force which causes the object to travel in the circular path. For example, when a car rounds an unbanked curve, the centripetal force is related to friction. Gravity provides the centripetal force for a satellite in orbit, while a ball on the end of a string is held in a circular path by the tension in the string.

TEXTBOOK QUESTION 1. Sometimes people say that water is removed from clothes in a spin dryer by centrifugal force throwing the water outward. What is wrong with this statement.

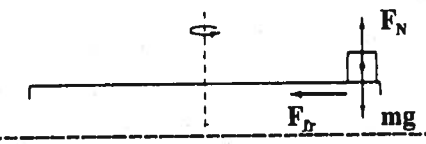
ANSWER: If the wall of the spinning cylinder was solid, then the inward force that the cylinder exerts on the water, i.e., centripetal force, would not allow the water to escape. However, the cylinder has numerous small holes in it and there is no centripetal force acting on the water at these holes. The inertia of the water in the wet clothes tends to cause the water to travel through these small holes. The water actually travels tangent to the circle in which the clothes are spinning and out through the holes. Centrifugal force has nothing to do with removing the water.

EXAMPLE PROBLEM 1. A 4.00 kg box is placed on the floor at the edge of a merry-go-round of radius 3.00 m. The coefficient of static friction between the box and the floor is 0.200. The merry-go-round accelerates from rest and eventually the box slides off the edge. Determine the speed at which this occurs.

Part a. Step 1.

Draw an accurate diagram locating all of the forces acting on the object.

Solution: (Section 5-2)



Part a. Step 2.

Complete a data table.

m = 4.00 kg $\quad \mu_s = 0.200 \quad$ r = 3.00 m $\quad$ v = ?

Part a. Step 3.

Determine the net force on the object.

The box is held in position by a frictional force that provides the centripetal acceleration. At the point where the box tends to slide off the circle, the force required to hold the box in position "just" exceeds the maximum possible value of the frictional force. The maximum value of the frictional force can now be determined.

$F_{fr} = \mu_s F_N$ but $F_N = mg$

$F_{fr} = \mu_s mg = (0.200)(4.00 \text{ kg})(9.80 \text{ m/s}^2)$

$F_{fr} = 7.84$ N and net F = 7.84 N

Part a. Step 4.

Use Newton's second law and the formula for centripetal acceleration to determine the object's velocity.

$\Sigma\vec{F} = m\vec{a}$ where net $\vec{F} = \vec{F}_{fr}$ and $a = v^2/r$.

$7.84 \text{ N} = (4.00 \text{ kg}) v^2/(3.00 \text{ m})$

$v^2 = 5.88 \text{ m}^2/\text{s}^2$ and $v = 2.42$ m/s

Note: it can be shown that the mass of the box does not affect the maximum velocity.

$F_{fr} = m v^2/r$ but $F_{fr} = \mu_s F_N = \mu_s mg$

$\mu_s mg = m v^2/r$

The mass cancels from both sides of the equation.

$\mu_s g = v^2/r$

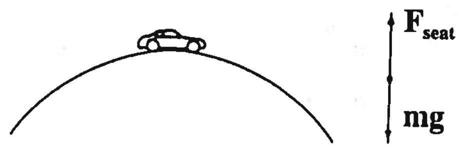
$v = (\mu_s g r)^{1/2} = [(0.200)(9.80 \text{ m/s}^2)(3.00 \text{ m})]^{1/2}$ and $v = 2.42$ m/s

EXAMPLE PROBLEM 2. A car travels over the crest of a hill at 10.0 m/s. The radius of curvature at the crest is 12.0 m. a) Determine the force exerted by the car seat on a 60.0 kg passenger. b) Determine the minimum speed required for the passenger to feel momentarily "weightless."

Part a. Step 1.

Draw a free-body diagram locating the forces acting on the passenger.

Solution: (Section 5-2)



Part a. Step 2.

Derive a formula for the net force acting on the person. In which direction is this force acting?

At the crest of the hill the car is traveling in a circle at a constant speed. Since it is traveling in a circle there is a centripetal acceleration directed into the center of the circle. This centripetal acceleration is produced by a net inward force equal to the difference between the passenger's weight, which acts downward, and the upward force exerted by the seat.

ΣF = weight - F_{seat} = mg - F_{seat}

Part a. Step 3.

Apply Newton's second law and the formula for centripetal acceleration and determine the magnitude of the force exerted by the seat.

Since the direction of the net force is downward, let the downward direction be positive.

$\Sigma F = m\,a$

weight - $F_{seat} = m\,a$ but $a = v^2/r$

$(60.0\ \text{kg})(9.80\ \text{m/s}^2) - F_{seat} = (60.0\ \text{kg})[(10.0\ \text{m/s})^2/(12.0\ \text{m})]$

$588\ \text{N} - F_{seat} = 500\ \text{N}$

$F_{seat} = 588\ \text{N} - 500\ \text{N} = 88.0\ \text{N}$

As the car goes over the crest of the hill, the passenger feels less upward force from the seat.

Part b. Step 1.

Determine the minimum speed required for the passenger to feel momentarily "weightless."

At this "minimum speed" the seat exerts no upward force on passenger, i.e., $F_{seat} = 0$. As a result the passenger experiences a feeling of momentary "weightlessness." Since the direction of the net force is downward, let the downward direction be positive.

$\Sigma F = m\ a$

weight - $F_{seat} = m\,a$ but $F_{seat} = 0$ and $a = v^2/r$

$(60.0\ \text{kg})(9.80\ \text{m/s}^2) - 0\ \text{N} = (60.0\ \text{kg})\ v^2/(12.0\ \text{m})$

$v = [(9.80\ \text{m/s}^2)(12.0\ \text{m})]^{1/2} = 10.8\ \text{m/s}$, or approximately 24 mph

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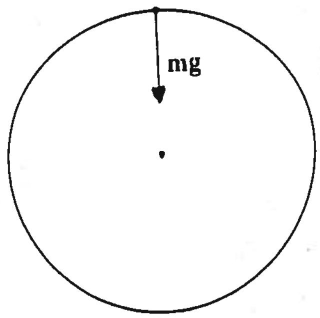
EXAMPLE PROBLEM 3. A student whirls a bucket of water in a vertical circle of radius 0.800 m. Determine the minimum a) speed and b) frequency of rotation required for the bucket to completely negotiate the top of the loop without water spilling out of the bucket.

Part a. Step 1. Draw an accurate diagram and locate the forces acting on the bucket at the top of the loop.	Solution: (Section 5-2) 
Part a. Step 2. Write a formula for the magnitude and direction of the net force acting on the water at the top of the circle.	If the bucket rotates with a speed greater than the minimum required to hold the water in the circle, then the bottom of the bucket exerts a downward force. Below the minimum velocity the water tends to fall out of the bucket. At the point of minimum speed, the bottom of the bucket exerts no force on the water when the bucket + water are at the top of the loop. The only force acting on the bucket + water is the force of gravity (mg). Therefore, net F = mg
Part a. Step 3. Apply Newton's second law and the concept of centripetal acceleration to determine the bucket's minimum speed.	$\Sigma F = ma$ but $a = a_R = v^2/r$ and from the previous step; net F = mg. Thus $mg = m\,v^2/r$ The mass of the bucket + water mathematically cancels from both sides of the equation. $g = v^2/r$ and upon simplifying $v^2 = r\,g = (0.800\text{ m})(9.80\text{ m/s}^2) = 7.84\text{ m}^2/\text{s}^2$ $v = 2.80$ m/s
Part b. Step 1. Determine the period of the motion.	Since the motion is uniform circular motion, $v = 2\pi r/T$ and $T = 2\pi r/v$ $T = 2\pi(0.800\text{ m})/(2.80\text{ m/s})$ and $T = 1.79$ s
Part b. Step 2. Determine the frequency of rotation.	The frequency (f) of rotation is inversely proportional to the period of the motion. $f = 1/T = 1/1.79\text{ s} = 0.557/\text{s}$ The frequency is 0.557 revolutions per second (0.557 rps or 0.557/s). It is convenient to express this in revolutions per minute (rpm); therefore, (0.57/s)(60 s/1 min) = 34 rpm.

Newton's Universal Law of Gravitation

The weight of an object placed on or very near the Earth's surface can be determined by using the equation weight = mg where g is approximately 9.8 m/s^2.

In general, the force (F) of gravitational attraction between any two objects of mass m_1 and mass m_2, their centers separated by a distance r, is given by **Newton's universal law of gravitation**: $F = G\, m_1 m_2/r^2$ where $G = 6.67 \times 10^{-11}\ N\ m^2/kg^2$ (the universal constant).

Gravity Near the Earth's Surface

For an object of mass m on or very near the Earth's surface,

$mg = G\, m\, m_e/r_e^2$ and $g = G\, m_e/r_e^2$

where m_e is the mass of the Earth and r_e the radius of the Earth. The acceleration due to gravity at some distance from the Earth can be determined by calculating the "effective" value of g. In order to determine the effective value of g, the radius of the Earth is replaced by the value for the radial distance between the object and the center of the Earth.

TEXTBOOK QUESTION 11. Does an apple exert a gravitational force on the Earth? If so, how large a force? Consider an apple a) attached to a tree and b) falling.

ANSWER: According to the universal law of gravitation, the force exerted by the apple on the Earth must be equal in magnitude to the force exerted by the Earth on the apple. According to Newton's third law, the forces must be equal but in opposite directions. The Earth pulls the apple downward while the apple pulls the earth upward.

In part a, the apple is attached to the tree and the apple's weight is balanced by an upward force exerted by the tree branch. However, it still exerts an upward force on the Earth.

In part b, the apple is falling and the net force on the apple equals the gravitational force exerted by the Earth. The apple exerts an equal but opposite force on the Earth. However, based on Newton's second law, the upward acceleration of the Earth is negligible because of its enormous mass.

TEXTBOOK QUESTION 21. Astronauts who spend long periods in outer space could be adversely affected by weightlessness. One way to simulate gravity is to shape the spaceship like a cylindrical shell that rotates, with the astronauts walking on the inside surface (see fig. 5-32). Explain how this simulates gravity. Consider (a) how objects fall, (b) the force we feel on our feet, and (c) any other aspects of gravity you can think of.

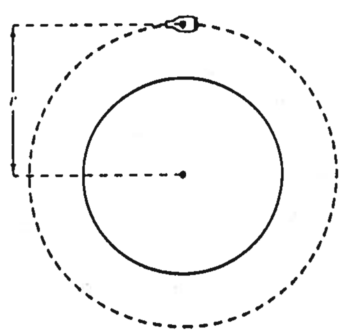
ANSWER: Let us assume that the shell rotates at constant speed and the astronaut is standing on the inner wall. The inner wall of the shell exerts an inward force, i.e. centripetal force, toward the center of the shell. Based on Newton's third law, for every action, there is an equal but opposite reaction. The inner wall would exert the action force on the astronaut's feet while the reaction force is the force the astronaut's feet exert on the inner wall. If the centripetal acceleration equals the gravitational acceleration experienced on Earth, then the centripetal force would equal the astronaut's weight on Earth.

If the astronaut dropped something, e.g., this book, the book would travel tangent to the circle in which it is traveling until it strikes the inner wall. However, as on Earth, the astronauts would see the book drop vertically downward towards his feet. This is because the astronaut is traveling approximately in the same circle as the book and at the same rate.

In a weightless environment, bones lose calcium and muscles atrophy. As a result, today's astronauts must spend several hours each day in vigorous exercise. The centripetal force provided by a rotating cabin would provide the artificial gravity needed to prevent physiological changes.

Another advantage would be the way an astronaut drinks liquids. Astronauts now must keep liquids in closed, squeezeable containers. This is because the liquids tend to float out of an open cup and form small droplets that fill the cabin. The astronaut could drink liquids from an open cup or container but, as on Earth, he would still have to careful not to knock the over the container.

EXAMPLE PROBLEM 4. On December 24, 1968, the Apollo VIII command module became the first manned vehicle to go into orbit above the surface of the Moon. Assuming that the orbit was approximately circular and the vehicle was 110 km (69 miles) above the lunar surface, determine the a) orbital velocity of the spacecraft and b) period of the motion.
Note: m_{Moon} = 7.40 x 10^{22} kg, r_{Moon} = 1.74 x 10^6 m, and 110 km = 1.10 x 10^5 m.

Part a. Step 1. Draw a diagram of the spacecraft's orbit about the Moon.	Solution: (Sections 5-7 and 5-8) 
Part a. Step 2. Determine the radius of the orbit as measured from the center of the Moon.	$r = r_{Moon}$ + altitude above surface = 1.74 x 10^6 m + 1.10 x 10^5 m r = 1.85 x 10^6 m
Part a. Step 3. What force holds the spacecraft in its circular orbit about the Moon?	Lunar gravity is the only force acting on the spacecraft and this force causes it to travel in uniform circular motion. $F = G\,m\,M/r^2$

Part a. Step 4. To solve for the orbital velocity, apply Newton's second law and the concept of centripetal acceleration.	$\Sigma F = F_{grav} = m\ a_R$ $G\ m\ M/r^2 = m\ v^2/r$ The mass of the orbiting object (m) appears on both sides of the equation and cancels algebraically. $v^2 = G\ M/r$ $= [(6.67 \times 10^{-11}\ N\ m^2/kg^2)(7.4 \times 10^{22}\ kg)]/(1.85 \times 10^6\ m)$ $v^2 = 2.7 \times 10^6\ m^2/s^2$ $v = 1.6 \times 10^3$ m/s; approximately 3600 mph
Part b. Step 1. Determine the period of the motion.	$v = 2\pi r/T$ $1.6 \times 10^3\ m/s = 2\ \pi\ (1.85 \times 10^6\ m)/T$ $T = 7.12 \times 10^3$ s or 1.97 hours

Kepler's Laws and Newton's Synthesis

Johannes Kepler used data collected by Tycho Brahe to empirically derive three laws for the motion of the planets about the Sun. These laws are now referred to as Kepler's laws of planetary motion and can be summarized as follows:

First law: The path of each planet about the Sun is an ellipse with the Sun at one of the focal points of the ellipse.

Second law: Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time.

Third law: The ratio of the square of the period (T^2) of any planet revolving about the Sun to the ratio of the cube of the planet's average distance from the Sun (r^3) is a constant, i.e., T^2/r^3 = constant. Also, the third law can be stated as the ratio of the squares of the periods of any two planets revolving about the Sun is equal to the ratio of the cubes of their average distances from the Sun, i.e., $T_1^2/T_2^2 = r_1^3/r_2^3$.

Newton was able to mathematically derive Kepler's laws by using his laws of motion and universal law of gravitation. Thus Kepler's laws, which were derived from empirical data, could be used to support Newton's universal law of gravitation.

TEXTBOOK QUESTION 23. The Earth moves faster in its orbit around the Sun in January than in July. Is the Earth closer to the Sun in January or in July? Explain. [Note: This not much of a factor in producing the seasons - the main factor is the tilt of the Earth's axis relative to the plane of its orbit.]

ANSWER: Based on Kepler's second law, the Earth moves faster in its orbit in January when it is closer to the Sun. The effect on the seasons due to the variation in the distance is minimal compared to the effect of the tilt of the Earth on its axis.

In July, the northern hemisphere is tilted toward the Sun. The number of hours of daylight is greater and the northern hemisphere receives more direct rays of the Sun. In January, the northern hemisphere is tilted away from the Sun. The Sun's rays are less direct and there are fewer hours of daylight. The combination of number of hours of daylight and the angle of the Sun's rays is the major cause of the variation in the seasons.

EXAMPLE PROBLEM 5. The mean distance from the Earth to the Sun is 1.496×10^8 km and the period of its motion about the Sun is 1.00 year. The period of Jupiter's motion about the Sun is observed to be 11.86 years. Determine the mean distance from the Sun to Jupiter.

Part a. Step 1.	Solution: (Section 5-8)
Complete a data table.	$T_1 = 11.86$ y $\quad r_1 = ?$ $T_2 = 1.00$ y $\quad r_2 = 1.496 \times 10^8$ km
Part a. Step 2.	$T_1^2/T_2^2 = r_1^3/r_2^3$
Use Kepler's third law to solve for the distance.	$(11.86\text{ y})^2/(1.00\text{ y})^2 = r_1^3/(1.496 \times 10^8\text{ km})^3$ $r_1^3 = (11.86)^2(1.496 \times 10^8\text{ km})^3 = 4.70 \times 10^{26}\text{ km}^3$ $r_1 = 7.8 \times 10^8$ km or approximately 480 million miles.

Types of Forces in Nature

Physicists now recognize only four different forces in nature. These are the gravitational force, electromagnetic force, strong nuclear force, and weak nuclear force. The electromagnetic force refers to electrical and magnetic forces. The strong nuclear force and weak nuclear force operate at the level of the atomic nucleus.

The ordinary pushes, pulls, and frictional forces that have been considered in previous chapters are referred to as "contact forces" because they involve objects that come into contact with one another, e.g., a boy pulling a wagon. According to modern understanding, contact forces are due to the electromagnetic force.

PROBLEM SOLVING SKILLS

Problems involving centripetal acceleration relate to Newton's second law of motion. Therefore, the steps followed in Chapter 4 of the textbook can be applied to this chapter. The following is an outline of the steps to be followed in attempting to solve problems related to uniform circular motion and centripetal acceleration:

1. Draw an accurate diagram locating each of the forces acting on the object in uniform circular motion. Take note of the force(s) that is/are causing the object to travel in a curved path.
2. Determine the magnitude and direction of the net force acting on the object.

3. Complete a data table based on information both given and implied in the statement of the problem.
4. Use Newton's second law and the concept of centripetal acceleration to solve the problem.

For problems involving Newton's universal law of gravitation:

1. Complete a data table based on information both given and implied in the problem.
2. Use the universal law of gravitation and, if necessary, Newton's second law and the concept of centripetal acceleration to solve the problem.

For problems involving Kepler's third law:

1. Complete a data table based on information both given and implied in the problem.
2. Use Kepler's third law to solve the problem.

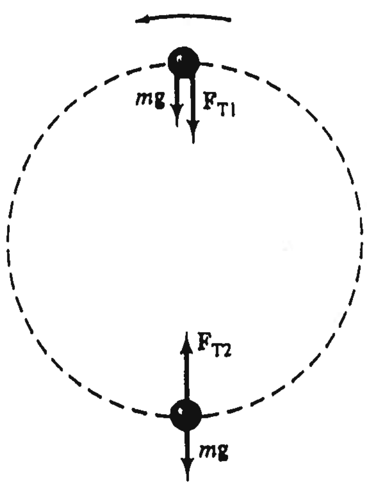
SOLUTIONS TO SELECTED TEXTBOOK PROBLEMS

TEXTBOOK PROBLEM 7. A ball on the end of a string is revolved at a uniform rate in a vertical circle of radius 72.0 cm, as shown in Fig. 5-33. If its speed is 4.00 m/s and its mass is 0.300 kg, calculate the tension in the string when the ball is (a) at the top of its path and (b) at the bottom of its path.

Part a. Step 1.

Fig. 5-33 locates the forces acting on the ball at the top and bottom of the loop.

Solution: (Section 5-2)



Part a. Step 2.

Complete a data table using information both given and implied.

r = 72.0 cm = 0.720 m v = 4.00 m/s m = 0.300 kg

$\vec{F}_{T1}$ = ? $\vec{F}_{T2}$ = ?

Part a. Step 3.

Determine the ball's centripetal acceleration.

The centripetal acceleration can be written in terms of the radius of the circle and the ball's velocity as follows:

$a_R = v^2/r = (4.00\ m/s)^2/(0.720\ m) = 22.2\ m/s^2$

Part a. Step 4.

Apply Newton's second law to determine the tension in the string at the top of the loop.

$\Sigma\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ where $\vec{\mathbf{a}} = \vec{\mathbf{a}}_R$

$F_{T1} + mg = m\,a_R$

$F_{T1} + (0.300\text{ kg})(9.80\text{ m/s}^2) = (0.300\text{ kg})(22.2\text{ m/s}^2)$

$F_{T1} + 2.94\text{ N} = 6.67\text{ N}$

$\vec{\mathbf{F}}_{T1} = 3.73\text{ N}$, downward

Part b. Step 1.

Apply Newton's second law to determine the tension in the string at the bottom of the loop.

$\Sigma\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ where $\vec{\mathbf{a}} = \vec{\mathbf{a}}_R$

$F_{T2} - mg = m\,a_R$

$F_{T2} - (0.300\text{ kg})(9.80\text{ m/s}^2) = (0.300\text{ kg})(22.2\text{ m/s}^2)$

$F_{T2} - 2.94\text{ N} = 6.67\text{ N}$

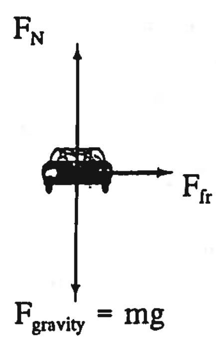
$\vec{\mathbf{F}}_{T2} = 9.61\text{ N}$, upward

TEXTBOOK PROBLEM 10. How large must the coefficient of static friction be between the tires and the road if a car is to round a level curve at of radius 85 m at a speed of 95 km/h?

Part a. Step 1.

Draw an accurate diagram locating all of the forces acting on the car.

Solution: (Section 5-2)



Part a. Step 2

Determine the car's speed in m/s.

$(95\text{ km/h})(1000\text{ m/km})(1\text{ h}/3600\text{ s}) = 26.4\text{ m/s}$

Part a. Step 3.

Complete a data table listing information both given and implied.

The road is level; therefore, the angle of the curve is 0°.

$m = ?$	$\mu_s = ?$	$\theta = 0°$
$r = 85\text{ m}$	$v = 26.4\text{ m/s}$	

Part a. Step 4.

Use Newton's second law, the frictional force, and the formula for centripetal acceleration to determine the coefficient of friction.

The car is held in the circle by a frictional force which provides the centripetal acceleration. At the point where the car tends to slide off the circle, the force required to hold the box in position "just" equals the maximum possible value of the frictional force.

$\Sigma F = m\,a$ and $a = v^2/r$

$F_{fr} = \mu_s F_N$ where $F_N = mg \cos\theta = mg \cos 0° = mg$

$F_{fr} = \mu_s\, mg$

The frictional force holds the car in the circle; therefore,

$\Sigma F = F_{fr}$

$m\, v^2/r = \mu_s\, mg$

Note: the mass cancels from both sides of the equation.

$\mu_s\, g = v^2/r$

$\mu_s = v^2/(rg) = (26.4\text{ m/s})^2/[(85.0\text{ m})(9.8\text{ m/s}^2)]$

$\mu_s = 0.84$

TEXTBOOK PROBLEM 15. How many revolutions per minute would a 15 m diameter Ferris wheel need to make for the passengers to feel "weightless" at the topmost point?

Part a. Step 1.

Use Fig. 5-9 in the textbook to determine the direction of the net force acting on the rider at the top of the loop.

Solution: (Section 5-2)

If the Ferris wheel rotates at a speed below the minimum speed to feel weightless, then there are two forces acting on the rider. The seat exerts an upward force (F_N) on the rider while gravity provides the downward force.

At the minimum speed to feel "weightless", gravity is the only force acting on the rider. The centripetal force is due to the rider's weight.

$\Sigma \vec{F} = m\,\vec{g}$

Part a. Step 2.

Determine the minimum speed in order to produce "weightlessness."

Since the Ferris wheel is traveling in a circle, the net force equals the centripetal force; therefore,

$m\, v^2/r = mg$ (Note that the mass cancels)

$v^2 = r\,g$ where $r = (15.0\text{ m})/2 = 7.5\text{ m}$

$v^2 = (7.5\text{ m}))(9.8\text{ m/s}^2) = 74\text{ m}^2/\text{s}^2$

$v = 8.6\text{ m/s}$

Part a. Step 4.

Determine the period of motion of the Ferris wheel.

$v = 2\pi\ r/T$ where T = period of the motion.

$T = 2\pi\ r/v = 2\ \pi\ (7.5\ m)/(8.6\ m/s) = 5.5\ s$

Part a. Step 5.

Determine the frequency of rotation in revolutions per minute (rpm).

$f = 1/T$

$f = [1/(5.50\ s)] = 0.18$ revolutions per second

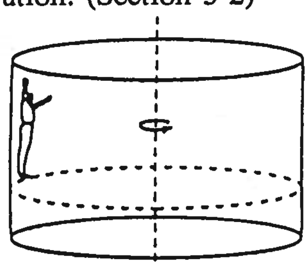
(0.18 revolutions/sec)[(60 s/1 min)] = 11 rpm

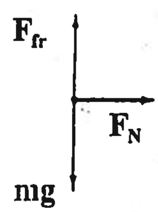
TEXTBOOK PROBLEM 18. In a "Rotor-ride" at a carnival, people are rotated in a cylindrically walled "room" (Fig. 5-35). The room radius is 4.6 m, and the rotation frequency is 0.50 revolution per second when the floor drops out. What is the minimum coefficient of static friction so that the people will not slip down? People on this ride say they were "pressed against the wall." Is there really an outward force pressing them against the wall? If so, what is its source? If not what is the proper description of their situation (besides scary)? [Hint: First draw the free-body diagram for a person.]

Part a. Step 1.

Draw a free-body diagram locating the forces acting on the rider.

Solution: (Section 5-2)





Part a. Step 2.

Determine the tangential speed of a person standing along the wall.

$v = 2\ \pi\ r/T = 2\pi\ r\ f$

$v = 2\ \pi\ (4.6\ m)(0.50\ rev/s)$

$v = 14\ m/s$

Part a. Step 3.

What forces must balance if the rider does not slip down the wall?

The rider does not slip down the wall; therefore, the upwardly directed frictional force must be equal but opposite to the rider's weight.

$\vec{F}_{fr} = m\ \vec{g}$

Part a. Step 4.

What force provides the radial acceleration?

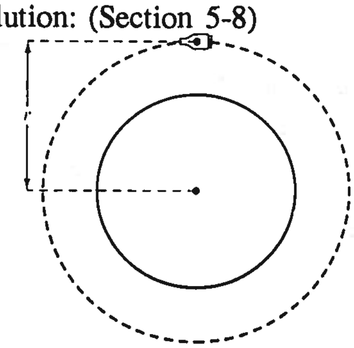
The net force on the rider is due to the normal force the wall exerts on the rider. This normal force is directed toward the center of circular motion and provides the centripetal or radial acceleration experienced by the rider.

$F_N = m\ v^2/r$

Part a. Step 5. Use Newton's second law and the concept of centripetal acceleration to solve for the minimum value for the coefficient of friction.	$F_{fr} = \mu_s F_N$ where $F_{fr} = mg$ and $F_N = m a_c = m v^2/r$ $mg = \mu_s m v^2/r$ Note: the rider's mass algebraically cancels. $g = \mu_s v^2/r$ and rearranging gives $\mu_s = r g/v^2 = (4.6 \text{ m})(9.8 \text{ m/s}^2)/(14 \text{ m/s})^2$ $\mu_s = 0.23$

There is no outward force "pressing" the rider against the wall. The rider tends to move tangent to the circle in which the room is traveling. The wall provides the centripetal force needed to keep the passenger moving in a circle. Based on Newton's third law, while the wall exerts an inward force on the rider, the rider exerts an outward force on the wall.

TEXTBOOK PROBLEM 43. Calculate the speed of a satellite moving in a stable circular orbit about the Earth at a height of 3600 km.

Part a. Step 1. Draw a diagram of the satellite's orbit about the Earth.	Solution: (Section 5-8) 
Part a. Step 2. Determine the radius of the orbit as measured from the center of the Earth. Note: 1 km = 1000 m.	The mass and radius of the Earth are given in inside front cover of the textbook. $m_{Earth} = 5.97 \times 10^{24}$ kg, $r_{Earth} = 6.38 \times 10^6$ m, $r = r_{Earth}$ + altitude above surface $r = 6.38 \times 10^6 \text{ m} + (3600 \text{ km})[(1000 \text{ m})/)1 \text{ km})]$ $r = 6.38 \times 10^6 \text{ m} + 3.60 \times 10^6 \text{ m} = 9.98 \times 10^6 \text{ m}$
Part a. Step 3. What forces are acting on the spacecraft?	The Earth's gravity is the only force acting on the spacecraft and this force causes it to travel in uniform circular motion. $F = G m M/r^2$

Part a. Step 4.

To solve for the orbital velocity, apply Newton's second law and the concept of centripetal acceleration.

$\vec{F}_{grav} = m\,\vec{a}_R$

$G\,m\,M/r^2 = m\,v^2/r$

The mass of the orbiting object (m) appears on both sides of equation and cancels algebraically.

$v^2 = G\,M/r$

$= [(6.67 \times 10^{-11}\ N\ m^2/kg^2)(5.97 \times 10^{24}\ kg)]/(9.98 \times 10^6\ m)$

$v^2 = 3.99 \times 10^7\ m^2/s^2$

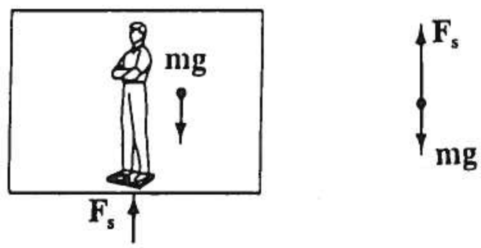
$v = 6.32 \times 10^3$ m/s; approximately 14,100 mph

TEXTBOOK PROBLEM 53. What will the spring scale read for a 55 kg student in an elevator that moves (a) upward with a constant speed of 6.0 m/s, (b) downward with a constant speed of 6.0 m/s, (c) upward with an acceleration of 0.33 g, (d) downward with an acceleration of 0.33 g, and (e) in free fall?

Part a. Step 1.

Draw a diagram locating the forces acting on the student.

Solution: (Sections 5-8)



Part a. Step 2.

Determine the reading on the scale when the elevator is traveling upward at a constant speed of 6.0 m/s.

The velocity is constant; therefore, the rate of acceleration is zero and the net force must equal zero. The student's apparent weight as read by the scale equals the actual weight.

$\Sigma F = m\,a$ and $F_s - mg = m\,a$ but $a = 0\ m/s^2$

$F - (55\ kg)(9.8\ m/s^2) = (55\ kg)(0\ m/s^2)$

$F = 540$ N

Part b. Step 1.

Determine the reading on the scale when the elevator is traveling downward at a constant speed of 6.0 m/s.

The velocity is constant; therefore, the rate of acceleration is zero and the net force must equal zero. The student's apparent weight as read by the scale equals the actual weight.

$F = m\,a$ and $F_s - mg = m\,a$ but $a = 0\ m/s^2$

$F - (55\ kg)(9.8\ m/s^2) = (55\ kg)(0\ m/s^2)$

$F = 540$ N

Part c. Step 1. Determine the scale reading when the elevator is accelerating upward at 0.33 g.	Since the acceleration is upward, then F_s is greater than mg and the net force is F_s - mg, directed upward. The student's apparent weight as determined by the scale is greater than the actual weight. Apply Newton's second law and determine the scale reading (F_s). $F = m\,a$ and $F_s - m\,g = m\,a$ $F_s - (55\text{ kg})(9.8\text{ m/s}^2) = (55\text{ kg})[(0.33)(9.8\text{ m/s}^2)]$ $F_s - 540\text{ N} = 180\text{ N}$ $F_s = 720\text{ N}$
Part d. Step 1. Determine the scale reading when the elevator is accelerating downward at 0.33 g.	Since the acceleration is downward, then F_s is less than mg and the net force is mg - F_s, directed downward. The student's apparent weight as determined by the scale is less than the actual weight. Apply Newton's second law and determine the scale reading (F_s). $F = m\,a$ and $mg - F_s = m\,a$ $(55\text{ kg})(9.8\text{ m/s}^2) - F_s = (55\text{ kg})[(0.33)(9.8\text{ m/s}^2)]$ $540\text{ N} - F_s = 180\text{ N}$ $F_s = 360\text{ N}$
Part e. Step 1. Determine the reading on the scale when it is in free fall.	When the elevator is in free fall, its downward rate of acceleration equals 9.8 m/s^2. $\Sigma F = m\,a$ and $mg - F_s = m\,a$ $(55\text{ kg})(9.8\text{ m/s}^2) - F_s = (55\text{ kg})(9.8\text{ m/s}^2)$ $F_s = 0\text{ N}$ The spring scale will not exert a force on the student. During free fall, the student will experience "apparent" weightlessness.